

Finite-size effects on the phase transition in a four- and six-fermion interaction model

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Abstract

We consider four- and six-fermion interacting models at finite temperature and density. We construct the corresponding free energies and investigate the appearance of first- and second-order phase transitions. Finite-size effects on the phase structure are investigated using methods of quantum field theory on toroidal topologies.

Key words: Phase transition, Finite-temperature field theory, Finite-size effects
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1 Introduction

Temperature and finite-size effects on the phase structure of physical systems can be described by quantum-field models defined in spaces with toroidal topologies. The starting point is the quantum theory of systems at finite temperature, both in the imaginary-time (Matsubara) [1,2] and real-time [3,4] formalisms. Since correlation functions should satisfy periodicity conditions on

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the time coordinate, known as Kubo-Martin-Schwinger (KMS) conditions, the finite-temperature theory is defined on the compactified manifold $\Gamma_4^1 = \mathbb{S}^1 \times \mathbb{R}^3$, where \mathbb{S}^1 is a circumference with length proportional to the inverse of the temperature and \mathbb{R}^3 is the Euclidean 3-dimensional space. Compactification of spatial dimensions [5,6] is considered in a similar way. An unified treatment, generalizing various approaches dealing with finite-temperature and spatial-compactification concurrently, has been constructed [7,8,9] by rigourously considering field theories on toroidal topologies $\Gamma_D^d = (\mathbb{S}^1)^d \times \mathbb{R}^{D-d}$, with $d (\leq D)$ being the number of compactified dimensions and D the dimension of the space-time.

These methods have been employed to investigate spontaneous symmetry-breaking induced by temperature and/or spatial constraints in some bosonic and fermionic models describing phase transitions in condensed-matter, statistical and particle physics; for instance, for describing the size-dependence of the transition temperature of superconducting films, wires and grains [10,11]; for investigating size-effects in first- and second-order transitions [12,13,14,15]; and for analyzing size and magnetic-field effects on the Gross-Neveu (GN) [16] and the Nambu-Jona-Lasinio (NJL) [17] models, taken as effective theories [18] for hadronic physics [19,20,21].

In this paper, we consider the massive GN model, modified by the inclusion of a $\eta(\bar{\psi}\psi)^3$ term in the Hamiltonian [22], at finite temperature and density. This model has been considered to investigate color superconductivity, with possible applications to neutron-star structure [23] and to chiral symmetry-breaking in hadronic systems [24]. Also, a rich phase structure, including BCS- and BEC-like phases, in strongly interacting matter has been obtained with an extended NJL model having six-fermion interaction [25]. Besides, such kind of interaction might be of relevance to systems in condensed matter where GN and NJL models are used, like graphene [26] and quantum phase transitions [27]. In all these cases, fluctuations due to finite-size play an important role in the phase structures.

Here, our aim is to discuss the finite-size effects in the extended GN model by considering compactification of spatial coordinates. We investigate the existence of first- and second-order phase transitions in this model and study their dependence on the relevant parameters: the temperature, chemical potential and the size of the system. We perform a study of the critical behavior of this model as a field theory on a toroidal space. We take the space-time dimension $D = 4$ and consider the particular case of two compactified dimensions ($d = 2$), imaginary time and one spatial coordinate. We find that, besides the usual symmetry restoration which occurs in the standard GN model, inverse symmetry-breaking happens in the extended model; in both cases, there exists a minimal size of the system below which the phase transition disappears.

2 The model

In a D -dimensional Euclidian manifold, \mathbb{R}^D , we consider the Hamiltonian for a modified massive GN model, which includes a six-fermion coupling [22,23,24],

$$H = \int d^D x \left\{ \psi^\dagger(x) (\gamma^j (i\partial_j)) \psi(x) - m'_0 \psi^\dagger(x) \psi(x) - \frac{\lambda_0}{2} [\psi^\dagger(x) \psi(x)]^2 + \frac{\eta_0}{3} [\psi^\dagger(x) \psi(x)]^3 \right\}, \quad (1)$$

where m'_0 , λ_0 and η_0 are the free-space physical mass and coupling constants at zero-temperature and zero-chemical potential. The γ -matrices are elements of the Clifford algebra and we use natural units, $\hbar = c = k_B = 1$. The mass parameter m'_0 may be taken as positive or negative, i.e. $m'_0 = \pm m_0$ with $m_0 > 0$, depending on the phase we choose to start (at zero temperature and chemical potential) and the nature of the phase transition.

From Eq. (1), we determine the finite temperature and density (chemical potential μ) corrections to the mass, considering one spatial dimension compactified in a circumference of length L ,

$$m(T, L, \mu) = m'_0 + \Sigma(T, L, \mu), \quad (2)$$

and to the coupling constants, $\lambda(T, L, \mu) = \lambda_0 + \Pi(T, L, \mu)$ and $\eta(T, L, \mu) = \eta_0 + \Xi(T, L, \mu)$. Then, a free-energy density of the Ginzburg-Landau type is constructed, as was considered in Ref. [28],

$$\mathcal{F} = \mathcal{F}_0 + A \phi^2(x) + B \phi^4(x) + C \phi^6(x), \quad (3)$$

where $A = -m$, $B = -\lambda/2$ and $C = \eta/3$. In this formalism, the quantity $\phi(x) = \sqrt{\langle \psi^\dagger(x) \psi(x) \rangle}$, where $\langle \cdot \rangle$ means thermal average in the grand-canonical ensemble, plays the role of the order parameter for the transition.

With our sign convention, to have a first-order transition, we must have $\lambda > 0$, $\eta > 0$ (to ensure stability of the system), with the mass m satisfying the condition $-16\eta m = 3\lambda^2$. In addition, for consistency, to have a first-order phase transition we must require that $m < 0$. On the other hand, we can recover a second-order transition for $\eta = 0$, but in this case we must have $\lambda < 0$ and the transition is characterized by $m < 0$ in the disordered phase and $m > 0$ in the ordered phase. We cannot have simultaneously $B < 0$ and $C = 0$ in Eq. (3), otherwise the stability of the system is lost. It is to be noted that the choice of $\pm m_0$ for the free-space mass at zero temperature and chemical potential corresponds to choosing whether we start with the system in the ordered ($+m_0$) or the disordered ($-m_0$) phase.

We shall consider the simplest approximation where the coupling constants are taken as fixed, i.e. $\lambda(T, L, \mu) = \lambda_0$ and $\eta(T, L, \mu) = \eta_0$, with temperature, chemical-potential and finite-size changes only appearing in the self-energy corrections of the mass term. Then, the free-energy density becomes

$$\mathcal{F} = \mathcal{F}_0 - m(T, L, \mu)\phi^2(x) - \frac{\lambda_0}{2}\phi^4(x) + \frac{\eta_0}{3}\phi^6(x). \quad (4)$$

Finite temperature and density corrections to the self-energy can be evaluated by using the Matsubara imaginary-time formalism. The Cartesian coordinates are specified by $x = (x_1 = \tau, x_2, \mathbf{x})$, where \mathbf{x} is a $(D - 2)$ -dimensional vector. The conjugate momentum of x is denoted by $k = (k_1, k_2, \mathbf{k})$, \mathbf{k} being a $(D - 2)$ -dimensional vector in momentum space. The KMS conditions, carrying the anti-periodicity for fermions, imply that the Feynman rules are modified by the well-known Matsubara prescription [7],

$$\int \frac{dk_1}{2\pi} \rightarrow \frac{1}{\beta} \sum_{n=-\infty}^{+\infty}, \quad k_1 \rightarrow \frac{2\pi}{\beta}(n + \frac{1}{2} - \frac{i\beta\mu}{2\pi}) \equiv \omega_n, \quad (5)$$

where ω_n are Matsubara frequencies and $\beta = T^{-1}$; this corresponds to the compactification of the imaginary time in a circumference of length β . We shall also investigate finite-size effects by considering the compactification of one spatial coordinate (x_2) in a length L using the generalized Matsubara prescription

$$\int \frac{dk_2}{2\pi} \rightarrow \frac{1}{L} \sum_{l=-\infty}^{+\infty}, \quad k_2 \rightarrow \frac{2\pi}{L}(l + \frac{1}{2}) \equiv \omega_l. \quad (6)$$

Here we have chosen anti-periodic boundary conditions for the spatial compactification, instead of the simpler periodic ones, because they emerge naturally in the generalization of the KMS conditions satisfied by correlation functions for fermionic fields in toroidal topologies [7,8].

We consider corrections to the mass to first order in the coupling constants. This means that there are only two relevant contributions to the self-energy, one proportional to λ_0 , and other proportional to η_0 ; these contributions correspond the “tadpole” and the “shoestring” diagrams shown in Fig. 1. Thus,

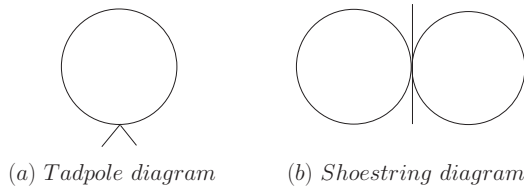


Fig. 1. Self-energy corrections to first-order in the coupling constants λ_0 and η_0 .

at the lowest order in the coupling constants, the finite temperature, chemical-

potential and size dependent self-energy is given by

$$\Sigma_D(\beta, L, \mu) = \Sigma_D^{(a)}(\beta, L, \mu) + \Sigma_D^{(b)}(\beta, L, \mu), \quad (7)$$

where $\Sigma_D^{(a)}(\beta, L, \mu)$ and $\Sigma_D^{(b)}(\beta, L, \mu)$ are respectively the contributions from the tadpole and the shoestring diagrams. We initially consider an arbitrary space-dimension D and later reduce to $D = 4$.

It should be noticed that the extended GN model is not perturbatively renormalizable in dimensions greater than two. However, it can be considered as an effective model and lowest-order calculations can be performed using an appropriate cut-off or, equivalently, a convenient minimal subtraction procedure, to discuss temperature, chemical-potential and size effects on its phase structure.

3 L -dependent self-energy at finite temperature and density

We now discuss the effects of finite temperature, density and size on the mass in order to construct the free-energy for the system.

3.1 *The contribution from the tadpole*

In the Euclidian space, the contribution from the tadpole diagram is given by

$$\Sigma_D^{(a)}(\beta, L, \mu) = -\frac{\lambda_0}{\beta L} \sum_{n,l=-\infty}^{\infty} \text{Tr} \int \frac{d^{D-2}k}{(2\pi)^{D-2}} \frac{m'_0 \mathbb{I}_D}{\mathbf{k}^2 + \omega_n^2 + \omega_l^2 + m_0^2}, \quad (8)$$

where the symbol Tr stands for the trace over spinor space. The minus sign in the above equation is due to the coefficient of ϕ^4 in Eq. (4). Defining the dimensionless quantities

$$a_1 = \frac{1}{(m_0\beta)^2}, \quad c_1 = \frac{1}{2} - \frac{i\beta\mu}{2\pi}, \quad a_2 = \frac{1}{(m_0L)^2}, \quad c_2 = \frac{1}{2}, \quad q_j = \frac{k_j}{2\pi m_0}, \quad (9)$$

for $j = 3, 4, \dots, D$, we write

$$\Sigma_D^{(a)}(\beta, L, \mu) = -\lambda_0 \frac{m'_0 m_0^{D-2}}{4\pi^2} \text{Tr} \mathbb{I}_D \sqrt{a_1 a_2} \mathcal{U}_2(s; a_j, c_j)|_{s=1}, \quad (10)$$

where the function $\mathcal{U}_2(s; a_j, c_j)$ is defined by

$$\mathcal{U}_2(s; a_j, c_j) = \sum_{n,l=-\infty}^{\infty} \int \frac{d^{D-2}q}{[\mathbf{q}^2 + a_1(n + c_1)^2 + a_2(l + c_2)^2 + b^2]^s}, \quad (11)$$

with $b = (2\pi)^{-1}$.

The integral appearing in the function $\mathcal{U}_2(s; a_j, c_j)$ can be treated with dimensional regularization techniques leading to

$$\mathcal{U}_2(s; a_j, c_j) = \pi^{\frac{D-2}{2}} \frac{\Gamma(\nu)}{\Gamma(s)} Y_2^{b^2}(\nu; a_j, c_j), \quad (12)$$

where $\nu = s - (D - 2)/2$ and

$$Y_2^{b^2}(\nu; a_j, c_j) = \sum_{n_1, n_2=-\infty}^{+\infty} \frac{1}{\left[\sum_{j=1}^2 a_j(n_j + c_j)^2 + b^2 \right]^\nu} \quad (13)$$

is a double-variable generalized Epstein-zeta function. It is to be noted that $Y_2^{b^2}$ is well-defined only for $\text{Re } \nu > 1$. However it can be analytically continued to the whole complex ν -plane, becoming a meromorphic function. The analytic continuation of $Y_2^{b^2}$ can be implemented through a generalized recurrence formula [29,30], leading to

$$Y_2^{b^2}(\nu; a_j, c_j) = \frac{\Gamma(\nu - 1)}{\Gamma(\nu)} \frac{\pi}{\sqrt{a_1 a_2}} (b^2)^{1-\nu} + \frac{4\pi^\nu b^{1-\nu}}{\Gamma(\nu) \sqrt{a_1 a_2}} \mathcal{R}_2(\nu; a_j, c_j), \quad (14)$$

where the regular part \mathcal{R}_2 is given by

$$\begin{aligned} \mathcal{R}_2(\nu; a_j, c_j) = & \sum_{j=1}^2 \sum_{n_j=1}^{\infty} \cos(2\pi n_j c_j) \left(\frac{n_j}{\sqrt{a_j}} \right)^{\nu-1} K_{\nu-1} \left(\frac{2\pi b n_j}{\sqrt{a_j}} \right) \\ & + 2 \sum_{n_1, n_2=1}^{\infty} \cos(2\pi n_1 c_1) \cos(2\pi n_2 c_2) \left(\sqrt{\frac{l_1^2}{a_1} + \frac{l_2^2}{a_2}} \right)^{\nu-1} \\ & \times K_{\nu-1} \left(2\pi b \sqrt{\frac{l_1^2}{a_1} + \frac{l_2^2}{a_2}} \right), \end{aligned} \quad (15)$$

with $K_\sigma(z)$ being the modified Bessel function of second kind.

For $s = 1$, we find that the first term in Eq. (14) diverges for even dimensions $D \geq 2$ due to the pole of the factor $\Gamma(\nu - 1)$. To obtain a finite tadpole contribution for the thermal self-energy at finite density with a compactified spatial coordinate, we perform a minimal subtraction by suppressing this polar

term and obtain, for $D = 4$,

$$\Sigma^{(a)}(\beta, L, \mu) = -\frac{2\lambda_0 m'_0 m_0^2}{\pi^2} \mathcal{W}_2(m_0\beta, m_0L, m_0^{-1}\mu), \quad (16)$$

where the function $\mathcal{W}_2(x, y, z)$ is given by

$$\mathcal{W}_2(x, y, z) = \mathcal{K}_1(x, z) + \mathcal{K}_1(y, 0) + \mathcal{K}_2(x, y, z), \quad (17)$$

while the functions \mathcal{K}_1 and \mathcal{K}_2 are defined by

$$\mathcal{K}_1(x, z) = \frac{1}{x} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cosh(xzn) K_1(xn), \quad (18)$$

$$\mathcal{K}_2(x, y, z) = 2 \sum_{n,l=1}^{\infty} \frac{(-1)^{n+l}}{\sqrt{x^2 n^2 + y^2 l^2}} \cosh(xzn) K_1\left(\sqrt{x^2 n^2 + y^2 l^2}\right). \quad (19)$$

Notice that the divergent part, which is subtracted, does not depend on β , L and μ , and so it does not interfere in the temperature, chemical-potential and size effects on the system. Also, it should be noted that the function \mathcal{K}_1 is negative for all values of $x > 0$, tends to 0 as $x \rightarrow \infty$ and it is quite insensitive to the value of z in the interval $0 \leq z < 1$, but it is not well defined, i.e. the series is not convergent, for $z \geq 1$. The behavior of the function \mathcal{K}_2 is similar.

3.2 The shoestring contribution and the self-energy

Using the dimensionless parameters defined in Eq. (9), the D -dimensional contribution of the shoestring diagram for the finite temperature and chemical-potential self-energy, with a compactified spatial dimension, can be written as

$$\Sigma_D^{(b)}(\beta, L, \mu) = 2\eta_0 \frac{m_0^{2(D-1)}}{16\pi^4} (\text{Tr } \mathbb{I}_D)^2 a_1 a_2 [\mathcal{U}_2(s; \{a_j\}, \{c_j\})]^2 \Big|_{s=1}. \quad (20)$$

It is to be noted that the shoestring diagram involves $m_0'^2 = m_0^2$ so that this contribution does not depend on the sign of m_0' . Repeating the steps described above and taking $D = 4$, we obtain the finite shoestring contribution as

$$\Sigma^{(b)}(\beta, L, \mu) = \frac{8\eta_0 m_0^6}{\pi^4} \left[\mathcal{W}_2(m_0\beta, m_0L, m_0^{-1}\mu) \right]^2. \quad (21)$$

Thus, the physical self-energy at finite temperature and density with a compactified spatial dimension, to first order in the coupling constants, becomes

$$\begin{aligned}\Sigma(\beta, L, \mu) = & -\frac{2\lambda_0 m'_0 m_0^2}{\pi^2} \mathcal{W}_2(m_0\beta, m_0L, m_0^{-1}\mu) \\ & + \frac{8\eta_0 m_0^6}{\pi^4} \left[\mathcal{W}_2(m_0\beta, m_0L, m_0^{-1}\mu) \right]^2.\end{aligned}\quad (22)$$

It should be noted that, in natural units, β and μ^{-1} have dimension of inverse of mass while the coupling constants λ_0 and η_0 have dimensions of $mass^{-2}$ and $mass^{-5}$, respectively, in $D = 4$. Since the function $\mathcal{W}_2(m_0\beta, m_0L, m_0^{-1}\mu)$ is dimensionless, $\Sigma(\beta, L, \mu)$ has dimension of mass, as it should.

Adding the contributions of the tadpole and the shoestring diagrams we obtain the (T, L, μ) -dependent mass, at first-order in the coupling constants λ_0 and η_0 ; using the reduced (dimensionless) parameters

$$t = \frac{T}{m_0} = \frac{1}{m_0\beta}, \quad \chi = \frac{1}{m_0L}, \quad \omega = \frac{\mu}{m_0}, \quad \lambda = \lambda_0 m_0^2, \quad \eta = \eta_0 m_0^5, \quad (23)$$

the corrected mass can be written as

$$m(T, L, \mu) = m'_0 - m'_0 \frac{2\lambda}{\pi^2} \mathcal{W}_2(t^{-1}, \chi^{-1}, \omega) + |m'_0| \frac{8\eta}{\pi^4} \left[\mathcal{W}_2(t^{-1}, \chi^{-1}, \omega) \right]^2. \quad (24)$$

It should be noticed that, due to the properties of the functions \mathcal{K}_1 and \mathcal{K}_2 , by taking the limit $L \rightarrow \infty$, which corresponds to considering the model at finite temperature and density in the free space, the corrected mass becomes

$$m(T, \mu) = m'_0 - m'_0 \frac{2\lambda}{\pi^2} \mathcal{K}_1(t^{-1}, \omega) + |m'_0| \frac{8\eta}{\pi^4} \left[\mathcal{K}_1(t^{-1}, \omega) \right]^2. \quad (25)$$

4 Finite-size effects on the phase transitions

We now discuss the effects of temperature, chemical-potential and finite-size on the mass of the system. Replacing the corrected mass, Eq. (24), into Eq. (4), we obtain the expression of the free energy density which should be analyzed to investigate the occurrence of phase transitions.

4.1 Finite-size effects on the second-order phase transition

In order to have a second-order phase transition in the system with one compactified dimension, we must take Eqs. (4) and (24) with $\eta_0 = 0$, $\lambda_0 < 0$ and $m'_0 = +m_0$, that is

$$\frac{m(t, \chi, \omega)}{m_0} = 1 + \frac{2|\lambda|}{\pi^2} \mathcal{W}_2(t^{-1}, \chi^{-1}, \omega). \quad (26)$$

For $\chi = 0$ and $\omega = 0$, corresponding to the model at free space and zero chemical-potential, it is easy to show that the coefficient of ϕ^2 in the free energy density, $-m(t, 0, 0)$, increases monotonically from the value -1 (for $t = 0$), vanishing at a critical temperature t_c , with $m < 0$ in the disordered phase above t_c . In the left panel of Fig. 2, the free-energy density, given by Eq. (4), is plotted as a function of the order parameter to illustrate the transition. Note that if one takes $m'_0 = -m_0$, the minimum of the free-energy density remains zero for all temperatures and no transition occurs.

For the system with one compactified dimension, from the critical condition $m(t, \chi, \omega) = 0$, for fixed values of the chemical potential and of the coupling constant, it follows that the critical temperature depends on the size of the system; this dependence is shown in the right panel of Fig. 2. We find that there is a minimal size of the system to sustain the broken-symmetry phase, which is independent of the chemical potential. However, this minimal size,

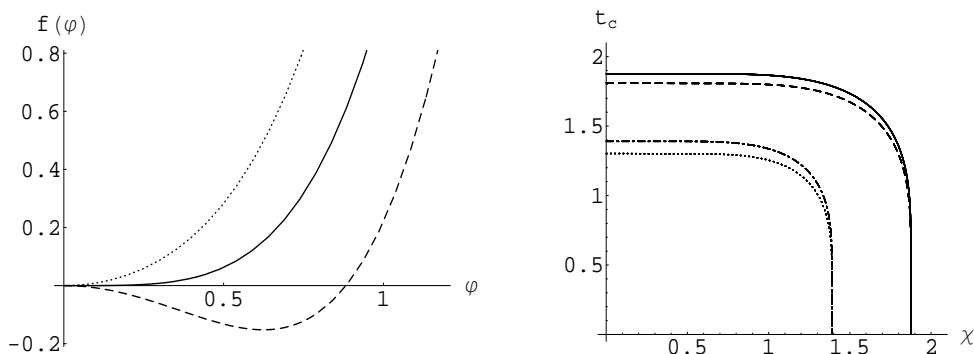


Fig. 2. Left panel: free energy density, $f(\phi) = (\mathcal{F}(\phi) - \mathcal{F}_0)/m_0^4$, as a function of the order parameter, $\phi = \phi m_0^{-3/2}$, for $\lambda = 2.0$, for the second-order phase transition in free space ($\chi = 0$), with fixed $\omega = 0.0$; dashed, solid and dotted lines correspond to $t = 1.0, 1.876$ and 2.5 , respectively. Right panel: reduced critical temperature of the second-order phase transition as a function of the reduced inverse size for two values of the chemical potential: $\omega = 0.0$ (solid line) and $\omega = 0.9$ (dashed line), for $|\lambda| = 2.0$; and, for the same values of ω , the dashed-dotted and the dotted lines, respectively, for $|\lambda| = 4.0$.

$L_0 = (m_0 \chi_0)^{-1}$, depends strongly on the strength of the quartic self-coupling.

4.2 Finite-size effects on the first-order phase transition

Taking the full Eq. (24), with $\lambda_0 > 0$, $\eta_0 > 0$ and fixing $m'_0 = -m_0$, there is a possibility that the system undergoes a first-order phase transition. In addition, it is required that the minimum values of the L -dependent free-energy density, Eq. (4), which occur for ϕ satisfying $\eta_0 \phi^5 - \lambda_0 \phi^3 - m\phi = 0$, should be equal to \mathcal{F}_0 , which can be fixed as zero without loss of generality;

this leads to the critical condition

$$-m(T, L, \mu) = \frac{3\lambda_0^2}{16\eta_0}. \quad (27)$$

This critical equation can be written in terms of dimensionless variables as,

$$\mathcal{G}_2(t, \chi, \omega; \xi) = \frac{\eta}{\lambda^2}, \quad (28)$$

where $\xi = \lambda/\eta$ and the function \mathcal{G}_2 defined by

$$\mathcal{G}_2(t, \chi, \omega; \xi) = \frac{3}{16} + \frac{2}{\pi^2} \frac{1}{\xi} \mathcal{W}_2(t^{-1}, \chi^{-1}, \omega) + \frac{8}{\pi^4} \left[\frac{1}{\xi} \mathcal{W}_2(t^{-1}, \chi^{-1}, \omega) \right]^2. \quad (29)$$

The behavior of the function \mathcal{G}_2 is presented in Fig. 3. One can see that the minimum value of \mathcal{G}_2 is $1/16$ while $\mathcal{G}_2(t = 0, \chi = 0, \omega; \xi) = 3/16$; thus, no solution of the critical equation (28) exists if $\eta/\lambda^2 < 1/16$, two solutions occur for $1/16 < \eta/\lambda^2 < 3/16$, while only one solution appears when $\eta/\lambda^2 > 3/16$.

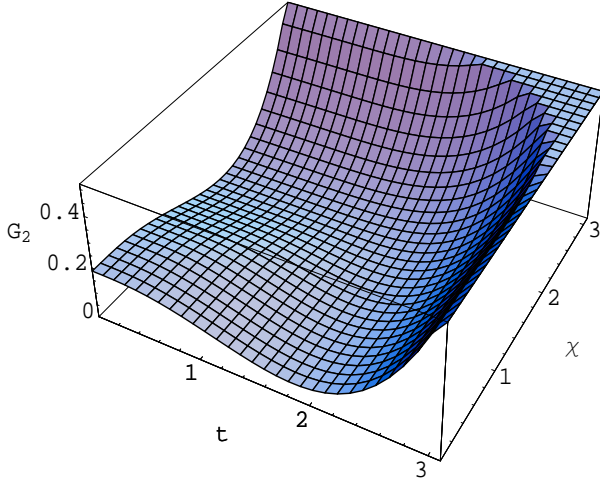


Fig. 3. Plot of \mathcal{G}_2 as a function of t and χ , for $\omega = 0.5$ and $\xi = 2.0$.

Critical curves in the (χ, t) - and (ω, t) -planes can be obtained simply by determining the level curves of the function \mathcal{G}_2 , at height η/λ^2 , for fixed values of the chemical potential and the inverse size. Phase diagrams for $\eta/\lambda^2 = 1/8$, corresponding to the case of a double-transition, are presented in the left panel of Fig. 4, taking two values of the chemical potential ω . For the single-transition case, the dependence of the critical temperature on the chemical potential, comparing bulk and finite-size systems, is shown in the right panel of Fig. 4.

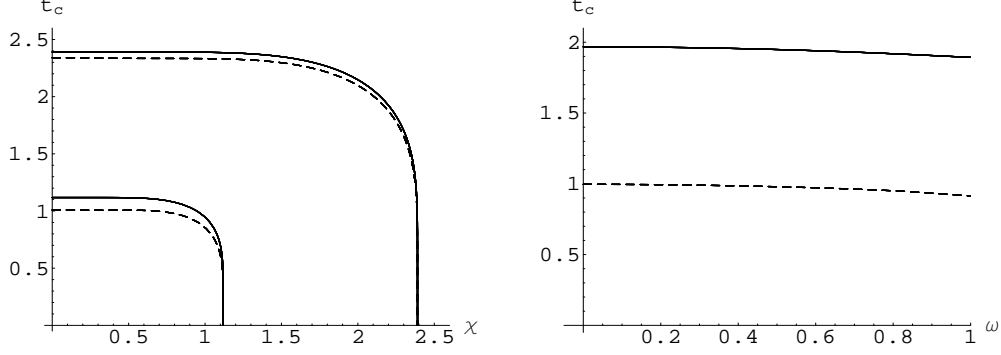


Fig. 4. Left panel: Phase diagrams of first-order phase-transitions in the (χ, t) -plane, for $\lambda = 4.0$ and $\eta = 2.0$; the solid and dashed lines correspond to $\omega = 0.0$ and $\omega = 0.9$, respectively. Right panel: Critical temperature as a function of the chemical potential for two values of χ : 0.0 and 1.96 (solid and dashed lines, respectively), taking $\lambda = 4.0$ and $\eta = 4.0$.

For the case of a double-solution (left panel of Fig. 4), analyzing the free-energy density, we find that, independently of the value of the chemical potential, the ordered (less symmetric) phase occurs for low temperatures and large sizes (the inner part of the smaller curve); in the intermediate region (between the curves), the stable phase is disordered; while, in the region outside the larger curve, the stable phase is an ordered one. In other words, the system reenters the ordered phase at high temperatures and/or small sizes, corresponding to a kind of inverse symmetry-breaking. This is illustrated in Fig. 5, for the system without spatial compactification.

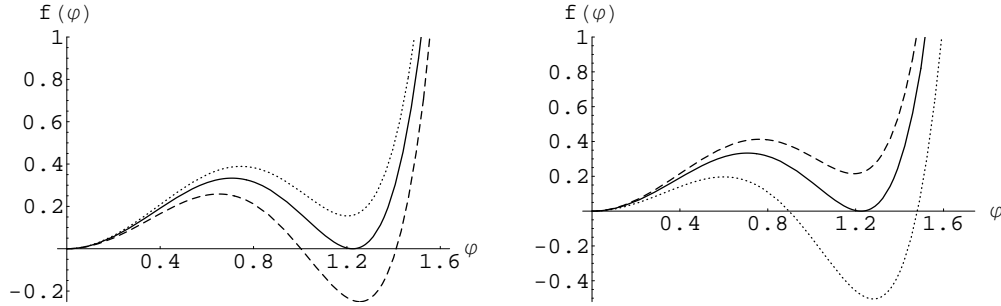


Fig. 5. The free energy density, $f(\varphi) = (\mathcal{F}(\varphi) - \mathcal{F}_0)/m_0^4$, as a function of the order parameter, $\varphi = \phi m_0^{-3/2}$, for $\eta = 2.0$ and $\lambda = 4.0$, for the first-order phase transition in free space ($\chi = 0$), with fixed $\omega = 0.5$. In the left panel, dashed, solid and dotted lines correspond to $t = 0.9, 1.085$ and 1.2 , and, in the right panel, they correspond to $t = 2.3, 2.374$ and 2.5 , respectively. The left panel corresponds to a symmetry restoration while the right panel shows an inverse symmetry breaking.

The whole phase structure depends on the values of the coupling constants but it is also a consequence of the sign chosen for the physical mass in free space, at zero temperature and chemical potential. For $\eta/\lambda^2 > 3/16$, there is only one critical line separating the disordered-phase (low t, χ) from the ordered

one (high t, χ), similar to the larger curves in the left panel of Fig. 4; this is a clear example of inverse-symmetry breaking. The transition temperature presents a weak dependence on the chemical potential, as shown in the right panel of Fig. 4; this happens in all situations, for both types of transition, and is a consequence of our approximation of neglecting corrections of coupling constants. Also, for all cases with $\eta/\lambda^2 > 3/16$ for which a first-order transition exists in the free space, there is a minimal length below which the phase-transition disappears, with the system remaining in the ordered phase for all temperatures.

It should be also pointed out that, as far as estimates of the transition temperature and the characteristic minimal size are concerned, these quantities depend on the coupling constants, the parameters of the model, and on the mass of the fermions, the natural mass scale of the model. However, independently of the values of these parameters, an exact relationship can be established between the free-space transition-temperature (T_c) and the characteristic minimal-size (L_0), for vanishing chemical-potential: due to the symmetry of the function $\mathcal{W}_2(t^{-1}, \chi^{-1}, \omega = 0)$ with respect to the variables t and χ , the reduced free-space transition-temperature, t_c , is identical to the reduced inverse minimal-size, χ_0 . In any case, when a first- or a second-order phase transition exists, there is a characteristic minimal size of the system below which the transition disappears. Actually, the chemical potential has little influence on the transition temperature, which decreases slightly as μ increases. Then we have in general $t_c \lesssim \chi_0$. Thus, for both types of phase-transitions and all values of the chemical potential, we can write, in natural units, $T_c L_0 \lesssim 1$.

5 Conclusions

We have investigated the appearance of phase transitions in a modified Gross-Neveu model, which includes a six-fermion interaction, in the four-dimensional Euclidian space-time. After constructing the corresponding free-energy density, we calculate corrections to the mass term due to finite size, finite temperature and density. This leads us to critical equations that yield, for an appropriate choice of the parameters, to a first- or a second-order phase transition.

For the standard GN model, not including the six-fermion interaction, we find a second-order phase transition with the critical temperature depending strongly on the strength of the quartic self-interaction. Considering the system with one compactified spatial dimension, we find that there is a minimal length below which the phase transition is suppressed and the system remains in the disordered phase for all values of the temperature; this characteristic length depends on the strength of the quartic self-interaction but it is independent

of the value of the chemical potential.

For the extended model, including a six-fermion self-interaction, we show that a first-order phase transition may exist, separating a low-temperature condensed phase from a high-temperature disordered phase, for a range of values of the coupling constants such that $1/16 < \eta/\lambda^2 < 3/16$. However, in this case, there is also a second branch of the transition line, as illustrated in the left panel of Fig 4, where the system reenters the condensed phase at even higher temperatures, a situation corresponding to an inverse symmetry-breaking. On the other hand, for $\eta/\lambda^2 > 3/16$ the transition occurs always in this unusual way, with the low- T phase being disordered and the high- T one being ordered, while for $\eta/\lambda^2 < 1/16$ no transition exist. In what the inverse symmetry-breaking is concerned, it is worth to mention that “exotic” phase transitions as these we find here, where the high temperature phase is less symmetric than the low- T one, are known in the literature. They resemble the inverse symmetry-breaking (or symmetry nonrestoration) which is found in some scalar multifield models [31,32,33,34,35,36]. Such unusual behavior has also been found in a gauge theory with an extra compactified dimension [37] and for the GN model with a random chemical potential [38]. Further work is needed to analyze the applicability of the points raised here and to discuss the effect of corrections to the coupling constants.

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